Voltage Level Detector

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THE VOLTAGE LEVEL DETECTOR

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BACKGROUND

The operation of a wide range of signal processing circuits depends on the ability to detect when an input voltage reaches some predetermined threshold value. A circuit designed to accomplish this detection is called a *voltage level detector* or *trigger* circuit. To be effective, the voltage level detector must provide an abrupt change in output whenever the input signal crosses the threshold value.

One way to accomplish the requirement of a large change in output voltage is to use an op-amp with *positive feedback* to the non-inverting (+) input. In this connection, the op-amp no longer operates in the linear mode. Rather, it will have an output that is either at the positive or at the negative saturation value, but not in between. Transition between these values occurs extremely fast whenever the inverting input voltage exceeds the threshold value.

GOALS

The purpose of this module is to study the use of an op-amp as a precise voltage level detector which produces an abrupt change in output whenever the input voltage reaches a predetermined threshold. Upon completion of the module the student should be able to:

- Explain the function of a voltage level detector in signal processing.
- Recognize an op-amp circuit connected for positive feedback.
- Describe how positive feedback is used in a voltage level detector.
- Calculate the threshold levels of an op-amp voltage level detector from circuit values.
- Build a voltage level detector using an op-amp.
- Measure the static and dynamic performance of a voltage level detector with and without offset and compare with expected values.

PREREQUISITES

To perform this module, the student should be familiar with the basic characteristics of the linear operational amplifier. He also must be familiar with the operation of basic electronic test equipment, including the DC voltmeter, sine wave generator and oscilloscope.

GLOSSARY

HYSTERESIS WIDTH - The range of input voltages in the voltage transfer characteristic curve bounded by the values where transitions in output voltage occur.

LINEAR MODE - A mode of operation where the op-amp output V_o is directly proportional to the input signal V_i. The proportionality constant, K, is the voltage gain of the amplifier.

$$V_o = KV_i$$

POSITIVE FEEDBACK - The application of the op-amp output signal to the non-inverting (+) input usually through a feedback resistor R_f.

SATURATION VALUE - An op-amp output voltage equal to the positive or negative power supply voltage ±V.

THRESHOLD VOLTAGE - The predetermined input voltage at which the voltage level detector output changes to the opposite stable state.

TRANSITION VOLTAGE - See Threshold Voltage.

TRIGGER VOLTAGE - See Threshold Voltage.

VOLTAGE TRANSFER CHARACTERISTICS - A graph of output voltage against input voltage.

discussion:

analyzing the circuits

VOLTAGE LEVEL DETECTOR

An operational amplifier can be used as a voltage level detector through the application of positive feedback to the non-inverting (+) input.

Positive Feedback:

To understand the operation of an op-amp with positive feedback, consider the circuit in Figure 1.

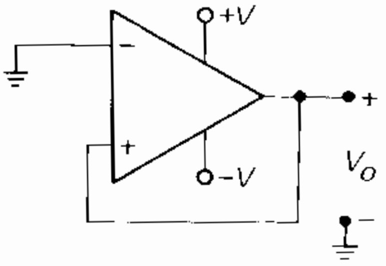


Figure I. AN OP-AMP WITH POSITIVE FEEDBACK is the heart of the voltage level detector. The reference voltage at the (+) input is equal to V_O.

Suppose that when the power is applied, the output V_o remains at exactly *zero* volts. Then it follows that the voltage between the (+) and (−) inputs is exactly zero since the output voltage must be proportional to the difference in the input voltages. In practice, however, it is difficult to keep the output voltage at *exactly* zero volts. If, for example, in Figure 1 the output drifts to a small positive value of V_o, because of feedback, this produces a still larger input until the output ultimately saturates at or near the positive supply voltage $+V$. Similarly, if the output voltage should initially drift slightly negative, then the effect of positive feedback is to drive the output to saturation at the negative supply voltage $-V$.

It is therefore the case that an op-amp with positive feedback has only two stable values of output voltage, the *positive* and *negative saturation* values which we can assume to be equal to the positive and negative supply voltages. Transitions between the two stable values take place very rapidly. Once a transition is triggered by the input signal, it goes to saturation without any further dependence on the input signal.

A more useful form of the voltage level detector is shown in Figure 2.

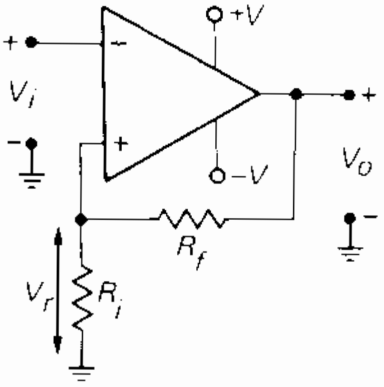


Figure 2. THE VOLTAGE LEVEL DETECTOR CIRCUIT. The divider R_f and R_i allows the reference voltage V_r to be some fraction of V_O.

Threshold Voltage:

As discussed above, the output voltage V_o has only two stable values, $+V$ and $-V$. The voltage at the non-inverting input also has only two stable values, which are related to the output voltage V_o through the voltage divider formed by R_f and R_i.

$$V_r = \frac{R_i}{R_f + R_i} V_o \qquad (1)$$

Rewriting the expression above in terms of the output saturation at the supply voltages $+V$ and $-V$:

$$V_r = \frac{R_i}{R_f + R_i}(+V) \qquad (2a)$$

$$\text{when } V_o = +V.$$

Or:

$$V_r = \frac{R_i}{R_f + R_i}(-V) \qquad (2b)$$

$$\text{when } V_O = -V.$$

A transition will take place whenever the input voltage V_i passes throught the voltage V_r in the direction which will cause the output to be driven to the opposite stable state. V_r is therefore called the *threshold* or *trigger* voltage for the voltage level detector.

Input voltage at which transitions in the output state will occur.

$$\boxed{V_i = \pm V_r} \qquad (3)$$

For example, if $R_i = 1k\Omega$, $R_f = 9k\Omega$ and $\pm V = 15V$, then from Equation (2a, 2b) the two stable states for V_r are $+1.5$ and $-1.5V$. Let us assume that $V_o = +15V$, $V_r = +1.5V$ and that V_i is less than $+1.5V$. As V_i increases and passes through the value $+1.5V$, the voltage *difference* between the (+) and (–) input changes from positive to negative, causing the output to switch abruptly to $-15V$. The voltage V_r now becomes equal to $-1.5V$, and this stable state lasts until V_i goes negative and passes through $-1.5V$. The output voltage then switches back to $+15V$. This variation of the output voltage as a function of the input voltage is shown graphically in Figure 3.

Hysteresis Width

Examination of Figure 3 shows that the positive-to-negative transition in the output does not occur at the same value of the input voltage as does the negative-to-positive output transition. This overlap of the stable states covers a range of input voltages that is centered around $V_i = 0$. It extends from $V_i = -R_i V/(R_f + R_i)$ on the negative side to $V_i = +R_i V/(R_f + R_i)$ on the positive side. Such dependence of the trigger point on the *direction* of the transition is called *hysteresis*. The *width* of the hysteresis region in Figure 3 is the difference between the two transition level and is equal to:

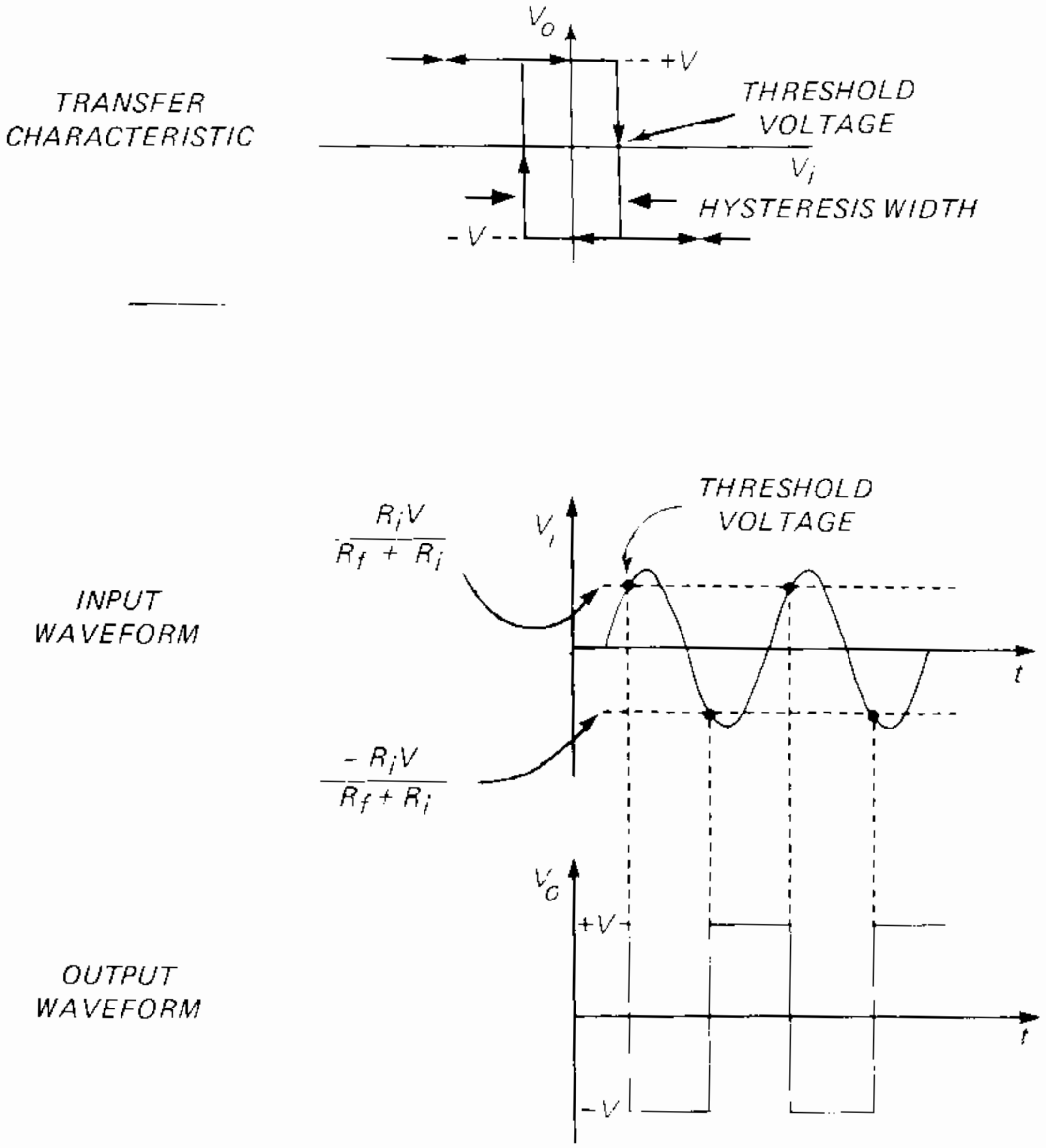


Figure 3. VOLTAGE TRANSFER CHARACTERISTIC of the voltage level detector of Figure 2 with typical input and output waveforms. The hysteresis is seen to be a result of the threshold voltage dependence on the direction of change in V_i.

Width of the overlap or hysteresis region in the voltage transfer characteristic

$$W = \frac{2R_iV}{R_f + R_i} \qquad (1)$$

When R_f [illegible] R_i, the hysteresis width is small and approximately proportional to [illegible]

The width can be reduced to zero by setting R_i to zero. In many applications a small amount of hysteresis is desirable because it provides some protection from noise fluctuations on the input signal V_i. Otherwise [illegible]

VOLTAGE LEVEL DETECTOR WITH DC OFFSET

An addition to the voltage level detector that adds versatility is shown in Figure 4.

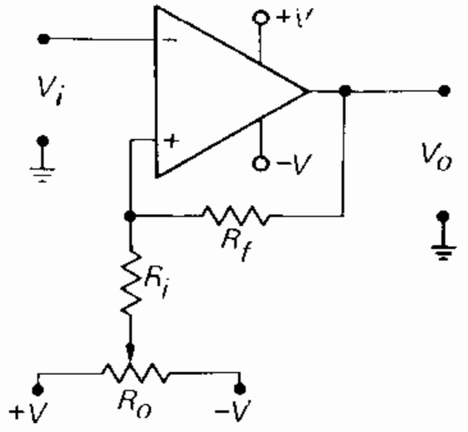


Figure 4. VOLTAGE LEVEL DETECTOR with voltage offset. The center voltage of the hysteresis region is determined by the setting of the pot R_o.

In this circuit the voltage V_r depends not only on the output voltage V_o but also on the setting of the potentiometer tap of R_o. The potentiometer arrangement acts just like a variable voltage source V_t which is adjustable from $-V$ through $+V$ volts with an internal resistance R_t that varies with the potentiometer setting.

The value of R_t will range between 0 and $R_o/4$ (see Appendix), with the maximum value occurring when the potentiometer is set at dead center. The entire voltage divider can therefore be modeled as a source V_t with R_t in series as shown in the dotted line of Figure 5.

Threshold Voltage:

As in the circuit of Figure 2, the transitions between the two stable output voltages will occur when V_i passes through the voltage V_r. In order to find the threshold voltage where triggering occurs, we must solve the circuit of Figure 5 for V_r. This is done in a derivation in the Appendix.

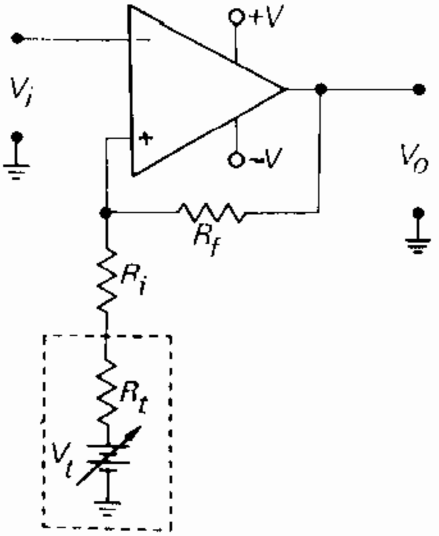


Figure 5. EQUIVALENT CIRCUIT of voltage level detector of Figure 4. The pot has been replaced by a variable voltage source (−V to +V) with a variable internal resistance (0 to R_o/4).

The results are:

$$V_i = \frac{(R_i + R_t)}{(R_f + R_i + R_t)}\ V + \frac{R_f}{(R_f + R_i + R_t)}\ V_t$$

Threshold voltage output equal to +V.

$$V_i = \frac{(R_i + R_t)}{(R_f + R_i + R_t)}(-V) + \frac{R_f}{(R_f + R_i + R_t)}\ V_t$$

Threshold voltage output equal to −V.

Hysteresis Width:

The resulting hysteresis width for the voltage level detector with offset is:

$$\boxed{W = \frac{2(R_i + R_t)\ V}{R_f + R_i + R_t}} \qquad (5)$$

Hysteresis width of the voltage transfer characteristic (with voltage offset).

This "dead" region is centered at the voltage V_c where:

$$\boxed{V_c = \frac{R_f V_t}{R_f + R_i + R_t}} \qquad (6)$$

Center voltage of hysteresis region.

Formulas (5) and (6) are derived in the Appendix. One possible transfer characteristic for this circuit is shown in Figure 6.

Note that the values of R_t and V_t depend on the setting of the potentiometer in Figure 4. However, in most cases we can choose R_f large compared to both R_i and R_t so that to a good approximation the center voltage becomes:

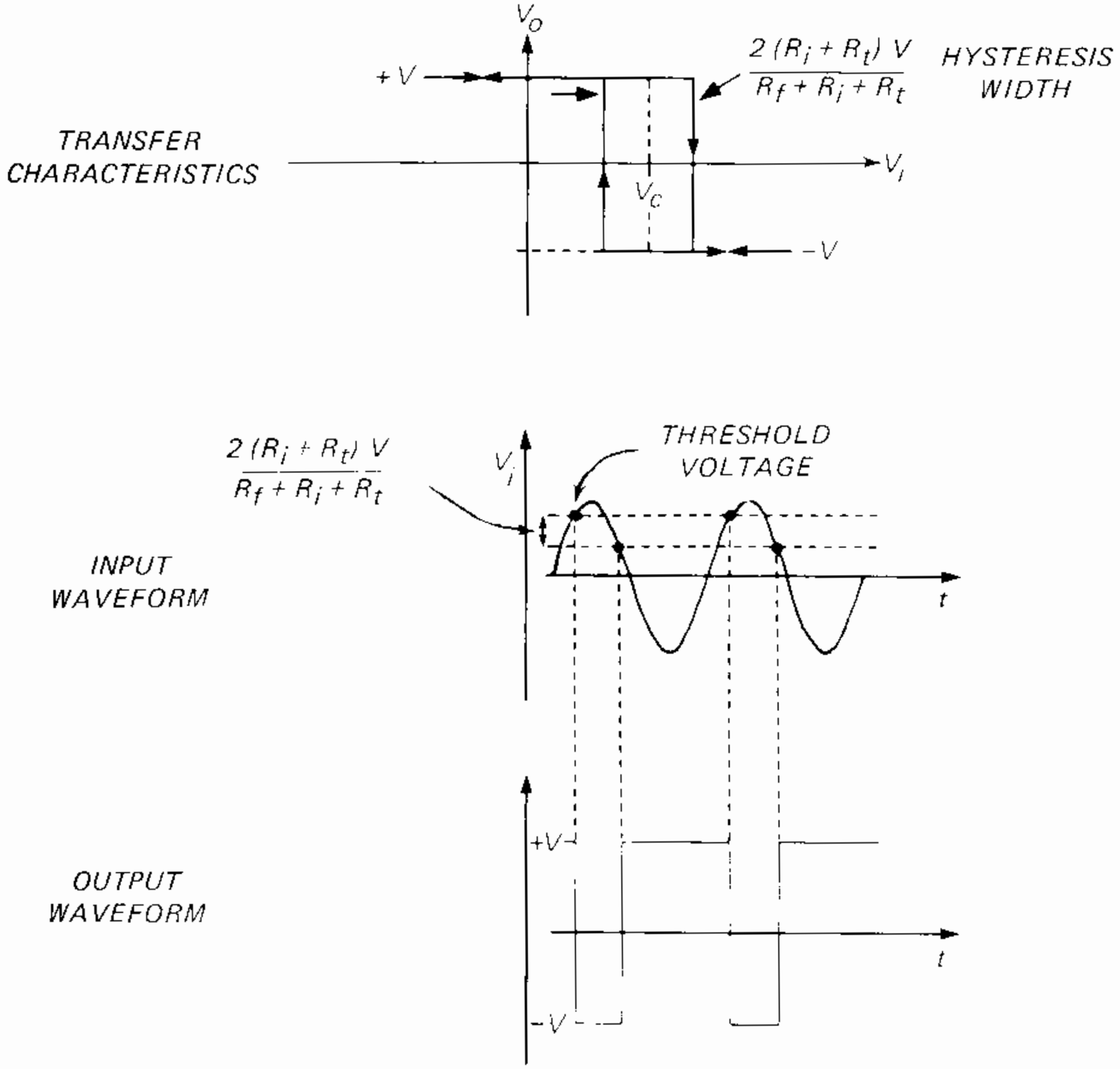


Figure 6. VOLTAGE TRANSFER CHARACTERISTICS of the voltage level detector of Figures 4 and 5 with typical input and output waveforms.

When R_f is much larger than R_i:

$$\boxed{V_c \simeq V_t} \qquad (7)$$

This value is simply the open-circuit voltage seen at the potentiometer tap, so the center voltage of the hysteresis region can be moved to any voltage between $-V$ and $+V$ by changing the potentiometer setting. The triggering voltage can therefore be adjusted to any desired value within those limits.

If, in the denominator of Equation (5), R_i and R_t are small compared to R_f, the hysteresis width becomes:

Hysteresis Width.

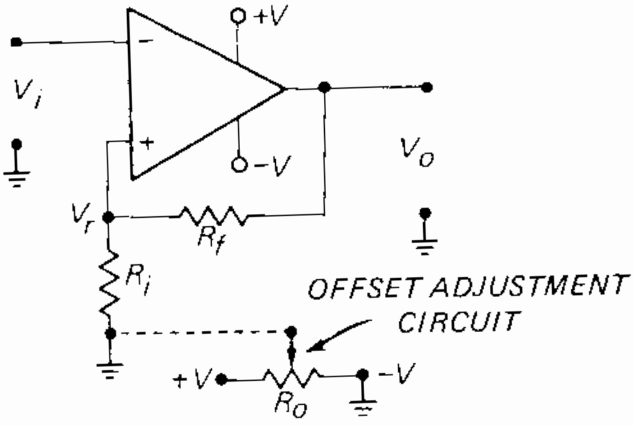
$$\boxed{W \simeq \frac{2(R_i + R_t)V}{R_f}} \qquad (8)$$

In order for the hysteresis width to be approximately independent of the potentiometer setting we require that R_i be much larger than $R_o/4$. This condition may not be so easily met, and so some hysteresis width change usually results when the triggering voltage is changed. If necessary this can be compensated for by making a corresponding change in R_i, but this is normally not necessary.

In summary, an op-amp circuit has been developed that is capable of being triggered at a voltage level which may be adjusted by setting a potentiometer. It is worth mentioning that this circuit is the same type as is used in the trigger circuit of an oscilloscope, with the level control on the scope control panel corresponding to the potentiometer R_o in Figure 4.

VOLTAGE LEVEL DETECTOR

CIRCUIT DESIGN INFORMATION



Voltage Level Detector with Offset:

V_r = Transition or triggering voltage-- Transitions between the two stable output voltages will occur when V_i passes through the voltage V_r developed across R_i:

$$\textit{Hysteresis Width} = \frac{2R_iV}{R_f+R_i}$$

$$V_r = \frac{R_i}{R_f+R_i}V \qquad \text{if } V_o = V$$

$$V_r = \frac{2R_iV}{R_f+R_i} \qquad \text{if } V_o = -V$$

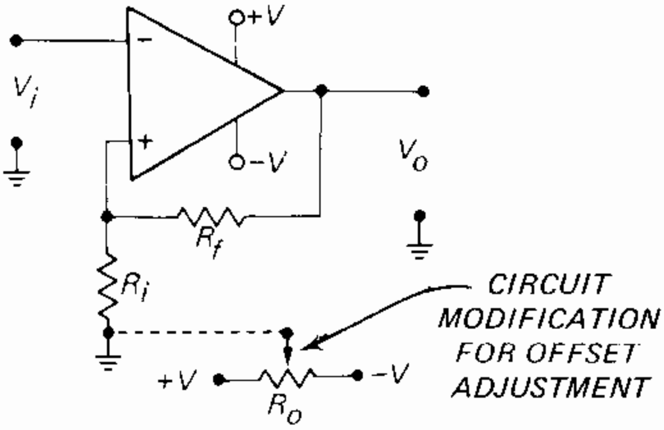
CIRCUIT PERFORMANCE DATA:

(Fill in component values used in actual circuit:)

R_i =

R_f =

R_o =



PARAMETER	*CIRCUIT CONDITIONS (Without Offset)*	*MEASURED VALUE*	*UNITS*
Transition Voltage:			
$V_r = \frac{R_i}{R_f + R_i} V$	$V_o = V$	V_r =	
$V_r = \frac{R_i}{R_f + R_i} V$	$V_o = -V$	V_r =	
Hysteresis Width:			
$\frac{2R_i V}{R_f + R_i}$		W =	
Output Voltage:			
$V_o - \pm V$	V_i *greater than* $+V_r$	V_o =	
	V_i *less than* $-V_r$	V_o =	

experiments :

measuring circuit performance

SUMMARY

In the following experiments the behavior of the voltage level detector is examined and its actual performance is compared with the theoretical predictions derived in the Discussion Section. In the first experiment the *static* characteristic is measured by varying the input voltage level and noting the points at which the output voltage makes a transition to the opposite stable state. The experimental values of the transition voltage and hysteresis width are compared with calculations based on circuit analysis.

A sine wave input signal is then used to study the *dynamic* characteristic of the voltage level detector. The effect of varying the hysteresis width is seen by examining the output waveform and the voltage transfer characteristic on the oscilloscope. Finally, an offset adjustment is included and the output waveform and voltage transfer characteristic are studied as a function of the hysteresis width and offset voltage adjustments.

The circuit values and experimental data for the voltage level detector should be recorded on the "CIRCUIT PERFORMANCE DATA" sheet on the opposite page.

MATERIALS AND EQUIPMENT

1	LEVEL DETECTOR breadboard with type 741 operational amplifier
1	Power Supply: ±12V to ±15V DC
1	DC Voltmeter (0-5 V)
1	Signal Generator: (400 Hz.)
1	Oscilloscope: Optional Dual Trace with External Horizontal Input

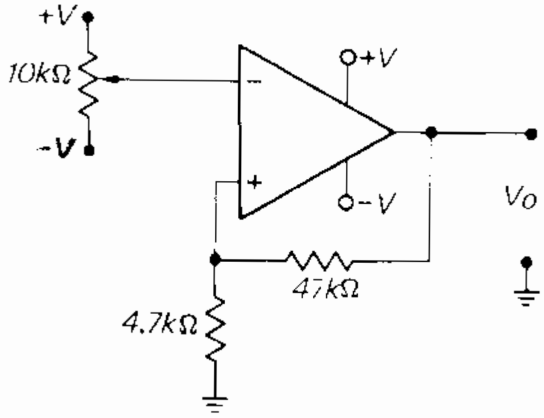
Fixed resistors:	1	47kΩ
	1	4.7kΩ
Potentiometers:	1	5kΩ
	1	10kΩ

VOLTAGE LEVEL DETECTOR

Static Characteristic:

1. *Build the voltage level detector circuit* shown in Diagram #1. The potentiometer connected to the inverting (-) input is used to provide a variable input voltage V_i of either polarity.

   NOTE: Be sure that the op-amp is inserted into its socket with the notch to the left.



CIRCUIT DIAGRAM #I

2. *Connect the power supply* to the circuit at +V and V.

3. *Connect an oscilloscope* (use DC coupling) to the output and connect a DC voltmeter between the inverting (-) input and ground.

4. *Set the potentiometer* so that the input voltage V_i is –5 volts.

   Measure the output voltage V_o with the oscilloscope.

5. *Increase the input voltage from –5 volts to +5 volts* and observe the output voltage transition.

   Record the approximate value of V_i at the time the transition takes place.

   Increasing $V_i \approx$ ____________ *V*

6. *Decrease* V_i *from +5 volts back to –5 volts* and again observe and record the approximate value of V_i at the time the transition takes place.

   Decreasing $V_i \approx$ ____________ *V*

7. *With the approximate values of* V_i at transition times known, fine tune the potentiometer to find the precise threshold value of V_i (in both directions).

   Record the values:

   Measured Triggering Voltages:

   V_i Increasing:

   $V_i\ trig =$ ______________________

   V_i Decreasing:

   $V_i\ trig =$ ______________________

8. *Calculate the triggering voltages* using Equations 2a and 2b, and compare the vlaues with the measured values.

   Calculated Triggering Voltages:

   V_i Increasing:

   $V_i\ trig = \frac{R_i}{(R_f + R_i)} V =$ ____________

   V_i Decreasing:

   $V_i\ trig = \frac{R_i}{(R_f + R_i)} (-V) =$ ____________

9. *Calculate the hysteresis width W.*

$W = V_i^{trig}$ *Increasing* - V_i^{trig} *Decreasing*

$W =$ ____________________ *Volts*

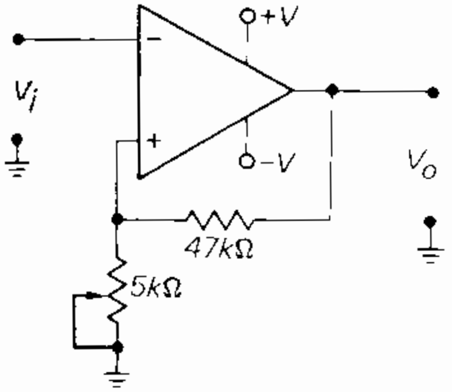
10. *Calculate the hysteresis width* from Equation 4 and compare with the measured value.

$$W = \frac{2R_i\, V}{(R_f + R_i)} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

11. *Disconnect the power supply.*

Dynamic Characteristic:

1. *Build the voltage level detector circuit* shown in Diagram #2. The 5kΩ pot is used to provide a variable resistance for R_i.



CIRCUIT DIAGRAM #2

2. *Connect the power supply* to the circuit at +V and −V and turn it on.

3. *Connect the oscilloscope* to the output of the level detector. Use a time base setting of 1 m sec/cm.

4. *Connect the signal generator* (sine wave) to the input of the level detector and set the frequency to *400 Hz*. Use a 5 volt peak-to-peak signal voltage.

5. *Set the 5kΩ potentiometer* so that the wiper is at the ground end.

6. *Measure and record both the frequency f and the amplitude A* of the output voltage V_O.

Output Voltage:

$f =$ ____________________

$A =$ ____________________

7. *Reduce the input voltage* until the level detector no longer triggers.

Measured Triggering Level:

$V_i^{trig} =$ ____________________

8. *Measure and record the highest input voltage* V_i at which triggering no longer occurs, using an oscilloscope.

9. *Carefully remove the 5k potentiometer* (R_i) from the circuit and measure the resistance.

Record the value.

$R_i =$ ____________________

10. *Replace the potentiometer* in the circuit and restore the input voltage to a 5 volt peak-to-peak level.

11. *Calculate the input triggering voltage* using Equations 2a and 2b.

Calculated Triggering Level:

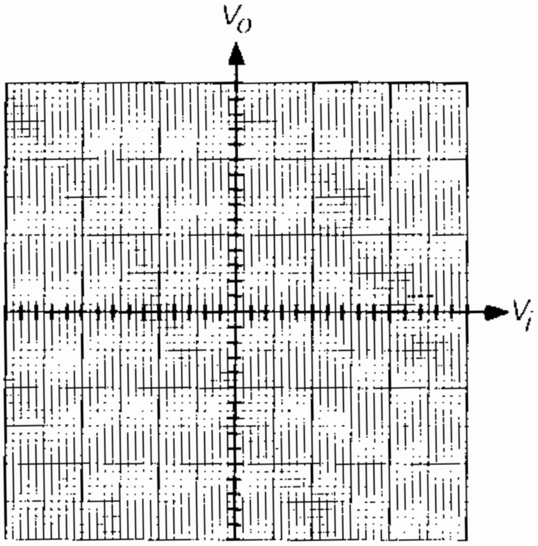
$$V_i^{trig} = \frac{\pm R_i}{(R_f + R_i)}\, V$$

$=$ ____________________

12. *Connect the external horizontal* (X-axis) *sweep input* of the oscilloscope to the *input* of the voltage level detector and switch the sweep control to external.

13. *Connect the vertical input* of the oscilloscope to the *output* of the voltage level detector.

    Sketch the resulting transfer characteristic on the scope display in the space provided.



SKETCH STEP #13

14. *Calculate the value of one horizontal division* given the horizontal sweep equal to the 5V peak-to-peak.

$$\text{VALUE PER DIV} = \frac{5\text{V P-P}}{\text{Total No. Of Divisions}}$$

$$= \underline{\qquad\qquad} \text{ Volts/Div.}$$

15. *Measure and record the hysteresis width W.*

    Minimum R_i:

    $W =$ ____________________ volts

16. *Increase the resistance* of the potentiometer and note the effect of changing R_i on the hysteresis width.

17. *Measure and record the hysteresis width* with the potentiometer set at its *maximum* value.

    Maximum R_i:

    $W =$ ____________________ volts

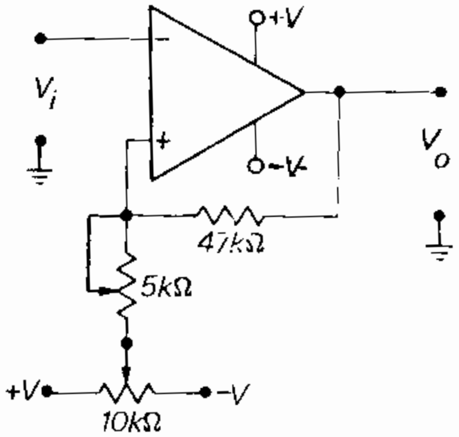
    Compare this with the widths calculated in steps 9 and 10.

18. *Disconnect the power supply.*

Voltage Level Detector With Offset

1. *Build the level detector* with offset as shown in Diagram #3.

   Set the 5kΩ and *10kΩ potentiometer* to approximately mid-range.



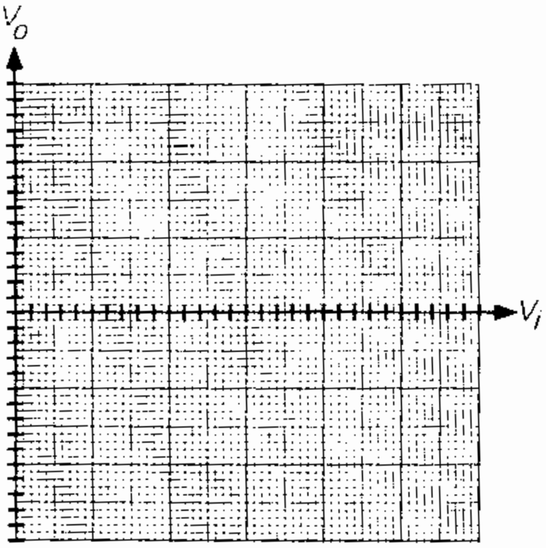
CIRCUIT DIAGRAM #3

2. *Connect the power supply* to the circuit at the points +V and −V and turn it on.

3. *Connect the signal generator* (sine wave) to the input of the level detector and set the frequency to *400 Hz*. Use a peak-to-peak input voltage which is *less* than the power supply voltage.

4. *Look at the output voltage V_O* with the oscilloscope. (DC coupling). *Sketch the waveform.*

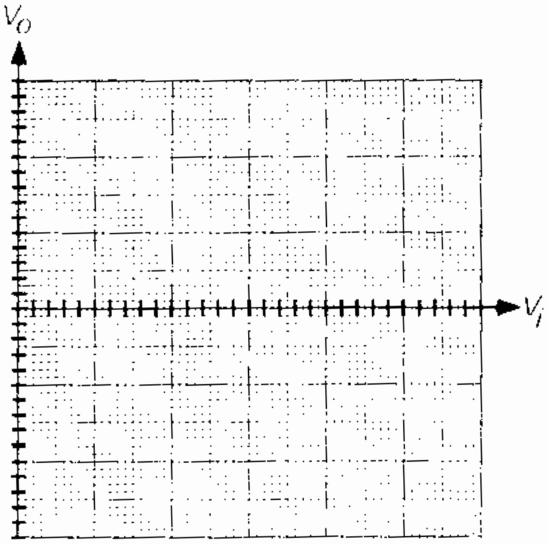
5. *Turn the 10kΩ offset pot* slowly clockwise and note the change in the waveform.



SKETCH STEP #4

Sketch the output waveform that appears just before a non-triggering state occurs.

NOTE: Refer to Figure 6 of the main text. If waveform is inverted, reverse the +V and −V connections on the 10kΩ potentiometer.



WITH OFFSET

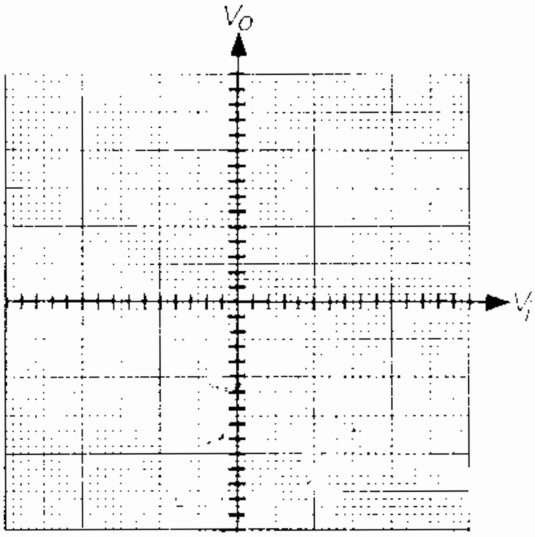
SKETCH STEP #5

6. *Re-set the 5kΩ* and *10kΩ* potentiometers to mid-range.

7. *Connect the external horizontal input* of the oscilloscope to the *input* of the level detector.

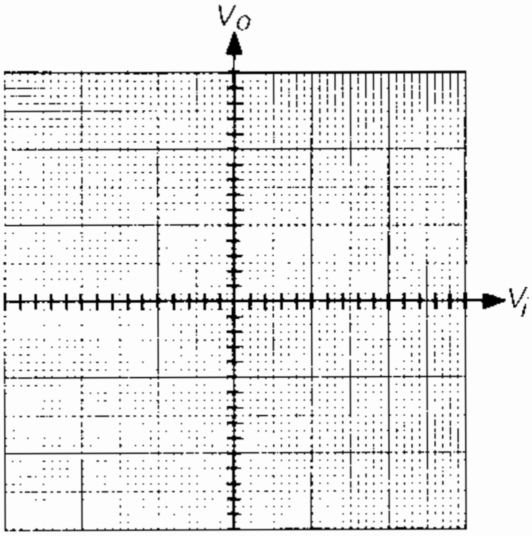
8. *Switch the sweep control* to external.

   Accurately sketch the resulting voltage transfer characteristic.



9. *Turn the 10kΩ offset potentiometer* slowly counter-clockwise and note the change in the waveform.

   Sketch the resulting transfer characteristic which appears just before the non-triggering state occurs.



SKETCH STEP #9

10. *Carefully note any change in the hysteresis width W* as the offset is varied, from zero to maximum value.

11. *Estimate and record the maximum percentage change in W* when the offset is varied.

Percentage Change In Hysteresis Width:

W% = ________________________%

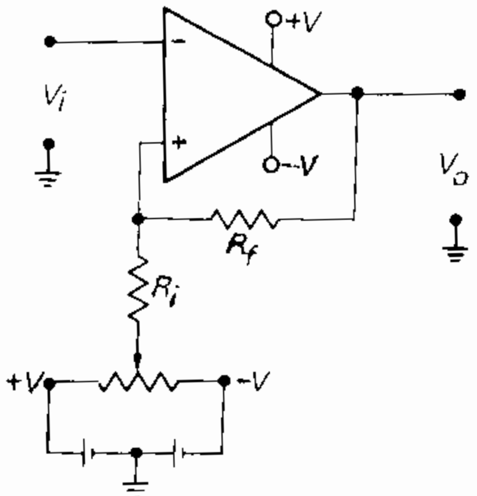
appendix:

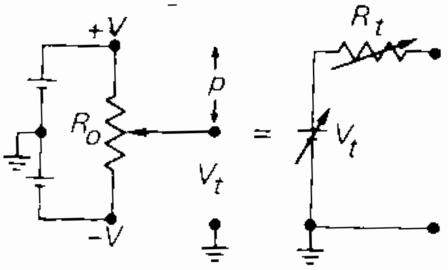
DERIVATION

Hysteresis Width and Center Voltage for the Voltage Level Detector with Offset:

The starting point for this derivation is the circuit of Figure 4. We first demonstrate that the model for the offset potentiometer used in Figure 5 is realistic. The total voltage drop across the potentiometer is equal to twice the supply voltage. But when measured with respect to ground, the open circuit voltage V_i at the wiper can lie anywhere between $-V$ and $+V$, and is equal to zero volts when the wiper is centered. The output resistance R_t of the potentiometer arrangement is equal to the total resistance of the path from the wiper point to ground.

If the pot has a resistance of R_O, and if we assume that the wiper cuts off p percent of the total resistance, then the path from the wiper to ground is formed by two parallel resistors, $(p-100)R_O$ in parallel with $(100-p)/100\ R_O$. The output resistance R_i is therefore equal to:





$$R_t = \frac{(\frac{p}{100}R_o)(\frac{(100-p)}{100}R_o)}{(\frac{p}{100}R_o + \frac{(100-p)}{100}R_o)}$$

$$= \frac{p}{100}\left(1 - \frac{p}{100}\right) R_O$$

Obviously, the value of R_t depends on the setting of the pot. When the wiper is at one end, say at 0% of the total R_O, then $R_t = 0$. Likewise if $p = 100\%$ (wiper at the other end), R_t again equals zero. The maximum value of R_t occurs when the wiper is exactly in the middle at $p = 50\%$. At this point:

$$R_t = 0.5\,(1 - 0.5)\,R_O$$

$$= \frac{1}{4}\,R_O$$

It is also worthwhile to set down how the open-circuit voltage V_t varies as the wiper is moved. The formula is:

$$V_t = \frac{p}{100}(2V) - V$$

$$= V\left(2\,\frac{p}{100} - 1\right)$$

As a check, note that when $p = 50\%$:

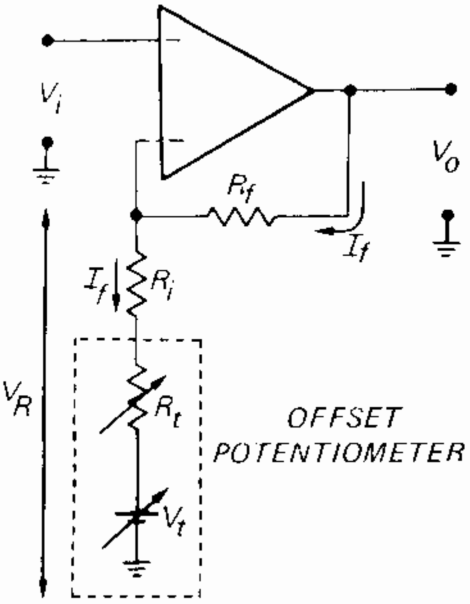
$$V_t = V\left(2 \cdot \frac{1}{2} - 1\right) = 0 \text{ volts}$$

The source model for the potentiometer thus allows us to draw the equivalent of the circuit in Figure 4, which is the same as Figure 5 in the text.

We assume that no current flows into or out of the non-inverting (+) input, so that the feedback current i_f must also flow through R_i and R_t.

Summing up the voltage drops around this loop yields:

$$V_O = I_f(R_f + R_i + R_t) + V_t$$



Or:
$$I_f = \frac{V_O - V_t}{(R_f + R_i + R_t)}$$

Now, the voltage V_r at the non-inverting (+) input is equal to:

$$V_r = I_f(R_i + R_t) + V_t$$

$$= \frac{(V_O - V_t)(R_i + R_t)}{(R_f + R_i + R_t)} + V_t$$

$$= \frac{(R_i + R_t)}{(R_f + R_i + R_t)}\,V_O$$

$$+ \frac{(R_f + R_i + R_t - R_i - R_t)}{(R_f + R_i + R_t)}\,V_t$$

$$= \frac{(R_i + R_t)}{(R_f + R_i + R_t)}\,V_O$$

$$+ \frac{R_f}{(R_f + R_i + R_t)}\,V_t$$

The triggering therefore occurs whenever $V_i = V_r$, and since the output voltage V_O has only two stable values ($+V$ and $-V$), this gives for the threshold level:

$$V_i = \frac{(R_i + R_t)}{(R_f + R_i + R_t)}(+V) + \frac{R_f}{(R_f + R_i + R_t)} V_t$$

$$\text{when } V_o = +V$$

$$V_i = \frac{(R_i + R_t)}{(R_f + R_i + R_t)}(-V) + \frac{R_f}{(R_f + R_i + R_t)}(V_t)$$

$$\text{when } V_o = -V$$

The hysteresis width W can again be found by taking the difference between the trigger level when $V_o = +V$ and the level when $V_o = -V$. This yields:

Hysteresis Width:

$$W = \frac{2(R_i + R_t)}{(R_f + R_i + R_t)} V$$

The common term in the two expressions for the triggering level represents the desired *offset* voltage. The "dead region" in the transfer characteristic is centered about the voltage V_c, where:

$$V_c = \frac{R_f}{(R_f + R_i + R_t)} V_t$$